NEUTRON SPECTRUM UNFOLDING USING THE LASSO REGRESSION METHOD

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TABLE OF CONTENTS

Page

ABSTRACT

Neutron Spectrum Unfolding Using
The Lasso Regression Method

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Neutron spectrum unfolding is used to adjust a known distribution in a reactor that is known to contain uncertainty. This known neutron spectrum is commonly determined in high resolution using transport codes such as MCNP. Measured reaction probabilities can be obtained through neutron activation analysis and can be used to adjust the simulated neutron spectrum. A linear system is defined that represents a relationship between the observed reaction probabilities and the calculated reaction probabilities that are a result of the simulated spectrum, and can be solved for an adjustment. The purpose of this investigation is to create a linear model with a simulated neutron spectrum and measured reaction probabilities, and to solve the model using the Lasso regression in the form of an elastic net regression. It uses benchmarks provided by the IAEA REAL-201X program to adjust the spectrum of the Annual Core Research Reactor in varying configurations to complete a proof of concept.

ACKNOWLEDGMENTS

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NOMENCLATURE

ACRR Annular Core Research Reactor

IAEA International Atomic Energy Agency

LASSO Least absolute shrinkage and selection operator

LB44 Lead-Boron 44

MCNP Monte Carlo N-Particle

PLG Polyethylene-Lead-Graphite

REAL Reaction Rates Estimates, Evaluated by Adjustment Analysis in Leading Laboratories

CHAPTER I

INTRODUCTION

An introduction will be provided that discusses the neutron spectrum of a research reactor and why it is both important and necessary to adjust this spectrum. This will include a brief summary of the current method used to obtain and adjust neutron energy spectra that is similar to the proposed method. Lastly, it will include a summary of the LASSO regression and why it has been selected for use in a proof of concept of spectrum adjustment.

Neutron energy spectra

The neutron energy spectrum represents the neutron distribution in a reactor as a function of energy. In a three-dimensional geometry, this energy spectrum is also a function of position. For the purpose of neutron irradiation experiments, it is of significance to know the neutron fluence at the point of material irradiation. There exists no such method of measurement that can determine this spectrum as a function of energy to the resolution desired.

As nuclear reactors consist of inhomogeneous materials, this neutron spectrum is often determined numerically using Monte Carlo simulation or by other deterministic transport methods. Within this investigation, MCNP (Monte Carlo N-Particle) is exclusively used in the determination of the simulated neutron energy spectrum. Monte Carlo methods in neutron transport simulate individual particles and record aspects of their average behavior. The average behavior of particles in the system is then inferred from the average behavior of the simulated particles. [1]

The issue is that the simulation of the neutron spectrum using a transport code such as MCNP is subject to uncertainty. These transport codes take many inputs including geometries, temperatures, densities, geometries, and other material properties. Within material properties there is also an assumption made for the state of reactor fuel burnup. In addition, there also exists uncertainty in the material cross-sections. Assuming that the transport code models the physics perfectly, it is clear that there still exists uncertainty in the final result that is the neutron spectrum.

For the purpose of this investigation, there will be no attempts to quantify this uncertainty. It is acceptable to only acknowledge that some uncertainty exists, and that the result is a neutron energy spectrum that is not perfectly correct.

Neutron spectrum adjustment

As it has been accepted that there exists uncertainty in the neutron energy spectra, there then exists a reason to determine a corrected spectra. The most common methods use information provided by activation foil measurements that are irradiated at the point of interest in the reactor. Knowing the material composition and reaction cross-sections of each activation foil, a simple relationship can be formed between the observed reaction probabilities and the integral contribution of the neutron flux within each energy group to the reaction rate. This relationship is

$$RP = \int_{E_{\min}}^{E_{\max}} \sigma(E)\phi(E)\, dE, \tag{I.1}$$

where RP is the deterined reaction activity for a foil, $E_{\min}$ is the minimum energy of the spectrum, $E_{\max}$ is the maximum energy of the spectrum, $\sigma(E)$ is the energy dependent reaction cross-section of the foil, and $\phi(E)$ is the neutron spectrum.

When using a differential spectrum binned into a structure of G energy groups with reaction cross-sections evaluated within each group, Eq. (I.1) can be approximated by

$$RP = \sum_{g=1}^{G} \sigma_g \phi_g \, \Delta E_g, \tag{I.2}$$

where σ_g is the reaction cross-section within energy group g, ϕ_g is the neutron flux within energy group g, and ΔE_g is the width of energy group g. This equation can then be replicated for multiple activation foil experiments. The problem of obtaining the individual ϕ's from this equation is frequently called the unfolding problem.

An ideal neutron energy spectrum contains a resolution of at least hundreds of energy groups. This implies that to perform an adjustment it is required to correct each energy group, or else there is great risk in creating a discontinuous neutron energy spectrum. This adjustment will be discussed further in the following chapter. The overall problem lies in the fact that there are commonly fewer activation foil measurements available than the number of energy groups. Therefore, the linear system being solved is severely underdetermined and has infinitely many solutions.

As a result of there being infinitely many solutions, intuition is relied on heavily to determine if the resulting adjusted spectrum is physically possible. There is a great interest in developing standards to enable an individual to determine which solution is correct, which is beyond the scope of this investigation. In addition, there will be no attempts to quantify the uncertainties in the resulting adjusted spectrum.

Least-squares methods of adjustment

The method of least squares minimizes the sum of the squares of the errors made in the results of a set of linear equations, which in this investigation is the relationship between measured activities of each foil and the integral contribution made by the neutron flux at each energy group. The majority of methods will solve a similar variant of the linear model produced in the following chapter. Iterative methods can then be used to tune the solution, as will also be done with the proposed method. [2]

The LASSO

The LASSO (least absolute shrinkage and selection operator) is comparable to least-squares methods, except it is a biased regression method that minimizes the least squares penalty plus a constant times the L1 norm of the coefficients. For a given set of N input measurements, denoted x_{i}, the LASSO solves

$$\min_{\beta_0,\beta}\left(\frac{1}{2N}\sum_{i=1}^{N}\left(y_i-\beta_0-x_i^T\beta\right)^2+\lambda\sum_{j=1}^{p}|\beta_j|\right), \tag{I.3}$$

where y_i is the response at observation i, and λ is a positive regularization parameter.

The benefit of this in terms of a neutron spectrum adjustment is that the LASSO regression is able to apply a penalty to the correction factors used in adjustment. This penalty term constrains the size of the estimated coefficients, which is beneficial when the coefficients are zero (suggesting no adjustment). As λ increases, the number of nonzero components of β decreases.

The `cv.glmnet` function is used in `R` to first run the regression to get a sequence of λ, and then multiple times to compute the fit with each of the folds omitted. The error is then accumulated,

and the average error and standard deviation over the folds is computed. The largest vlaue of λ is then chosen such that the error is within 1 standard deviation of the minimum.

Elastic net

The elastic net is a reguarlized regression that linearly combines the L1 and L2 penalties of the LASSO and ridge regressions. The significance of this is that elastic net overcomes the limitations of the LASSO, in which it tends to select at most only a few variables in a set of high-dimensional data with few examples. It does this by adding a quadratic part to the penalty, which when used alone is ridge regression.

For a given set of N input measurements, denoted x_i, the elastic net solves

$$\min_{\beta_0,\beta}\left(\frac{1}{2N}\sum_{i=1}^{N}\left(y_i-\beta_0-x_i^T\beta\right)^2+\lambda\sum_{j=1}^{p}\left(\frac{1-\alpha}{2}\beta_j^2+\alpha|\beta_j|\right)\right), \tag{I.4}$$

where α is a user-defined parameter. When $\alpha = 1$, elastic net is the same as LASSO. As it shrinks towards 0, it approaches ridge regression. For values of α between 0 and 1, the penalty term interpolates between the L1 and L2 norm of β. It is possible then to optimize elastic net by iterating through values of α and examining the results. A simple method of optimization will be presented in the following chapter.

CHAPTER II

METHODOLOGY

The input

The first set of inputs to this method will be a simulated, differential neutron energy spectrum binned with the desired energy group structure. The second set of input will be a set of measured specific activities of activation foils irradiated at the same location as the simulated spectrum. Reaction cross-sections will be input for the irradiated foil materials. There exist methods that can be used to create a new energy structure if the structure of the neutron spectrum and cross-sections differs. For the sake of simplicity, the cross-sections are assumed to be binned with the same energy group structure as the simulated neutron spectrum.

The linear model

First, a relationship between the measured specific activities and the simulated spectrum must be determined as summarized in the introduction. For an irradiated foil with reaction probability *RP*, this relationship is

$$RP = \int_{E_{\min}}^{E_{\max}} \phi(E)\sigma(E)\, dE, \tag{II.1}$$

where $E_{\min}$ is the minimum energy of the simulated spectrum, $E_{\max}$ is the maximum energy of the simulated spectrum, $\sigma(E)$ is the reaction cross-section of the irradiated foil, and $\phi(E)$ is the differential spectrum.

The reaction probability can be related to the measured reaction by

$$RP = \frac{A_s}{\lambda}, \tag{II.2}$$

where A_s is the measured specific activity and λ is the decay constant of the product nuclide.

As the simulated differential spectrum is binned into a structure G energy groups, where the spectrum is assumed constant across an energy group, the reaction probability can then be approximated as

$$RP = \sum_{g=1}^{G} \phi_g \sigma_g \Delta E_g, \tag{II.3}$$

where σ_g is the reaction cross-section of the foil within energy group g, ϕ_g is the simulated scalar flux within energy group g, and ΔE_g is the width of energy group g.

As the goal in this investigation is to adjust the simulated spectrum, it is necessary to define a correction factor, ω_g, for each energy group g. The correction factors will be defined such that a correction factor of 0 implies no adjustment, and a correction factor of ± 1 implies an adjustment of $\pm 100\%$. Eq. (II.3) will then be modified such that

$$RP = \sum_{g=1}^{G} \left(1 + \omega_g\right) \phi_g \sigma_g \Delta E_g. \tag{II.4}$$

The re-organization of the above equation in a format acceptable for a linear model is then

$$RP - \sum_{g=1}^{G} \phi_g \sigma_g \Delta E_g = \sum_{g=1}^{G} \omega_g \phi_g \sigma_g \Delta E_g. \tag{II.5}$$

Lastly, the complete linear model will be defined with F foil and cover combinations as

$$RP_i - \sum_{g=1}^{G} \phi_g \sigma_{g,i} \Delta E_g = \sum_{g=1}^{G} \omega_g \phi_g \sigma_{g,i} \Delta E_g, \quad \text{for } i = 0, 1, \ldots F, \tag{II.6}$$

where RP_i is the reaction probability of foil i, and $\sigma_{g,i}$ is the reaction cross section of foil i within energy group g.

Normalization

It is necessary to normalize the separate reactor operations to a uniform neutron fluence. Therefore, one reaction probability is used to normalize the remaining data. For the sake of simplicity, this reference foil will be the foil denoted with the index $i = 1$. A constant, c, is then defined such that

$$RP_1 - \sum_{g=1}^{G} c \phi_g \sigma_{g,1} \Delta E_g = 0, \tag{II.7}$$

therefore

$$c = \frac{RP_1}{\sum_{g=1}^{G} \phi_g \sigma_{g,1} \Delta E_g}. \tag{II.8}$$

This will result in the final linear model of

$$RP_i - \sum_{g=1}^{G} c \phi_g \sigma_{g,i} \Delta E_g = \sum_{g=1}^{G} \omega_g c \phi_g \sigma_{g,i} \Delta E_g, \quad \text{for } i = 1, 2, \ldots F. \tag{II.9}$$

Forward difference

A first-order forward difference method will be used to compare the continuity of the simulated spectrum and the resulting adjusted spectrum. The general first-order forward finite difference formula is of the form

$$f'(x) = \frac{f(x+h) - f(x)}{h}. \tag{II.10}$$

Knowing this, the first-order forward difference approximation of the neutron spectrum within energy group g can be defined as

$$\phi'_g = \frac{2(\phi_{g+1} - \phi_g)}{\Delta E_g + \Delta E_{g+i}}, \tag{II.11}$$

where ϕ_{g+1} is the neutron flux within energy group $g+1$, and ΔE_{g+i} is the width of energy group $g+1$.

The summation of all of the forward differences at each spectral point within the simulated spectrum can then be compared to the summation of all of the forward differences at each spectral point within the adjusted spectrum in order to roughly quantify the change in continuity of the adjusted spectrum. The following ratio, R, is then defined using this comparison

$$R = \frac{\sum_{g=1}^{G-1} \phi'_{\text{spec},g}}{\sum_{g=1}^{G-1} \phi'_{\text{adj},g}}, \tag{II.12}$$

where $\phi'_{\text{spec},g}$ is the first-order forward difference of the simulated spectrum within energy group g, and $\phi'_{\text{adj},g}$ is the first-order forward difference of the adjusted spectrum within energy group g. This ratio allows for a simple numerical comparison between multiple adjusted spectra, which will later be used to optimize the elastic net parameter.

Execution

The linear model represented in Eq. II.9 will be defined and the regression will be preformed using the `glmnet` package in the `R` language, which is a language and environment for statistical computing. The forward difference method as presented above will be used to optimize for the elastic net alpha parameter, in hopes of avoiding discontinuity in the adjusted spectrum.

CHAPTER III

RESULTS

The Annular Core Research Reactor

The data used has been published by Sandia National Laboratory for the Annual Core Research Reactor (ACRR) configured in the free field environment, the LB44 environment, and the PLG bucket environment. The data has been published as a part of the IAEA Neutron Spectral Adjustment Exercise REAL-201X, which is a project with the purpose of validating the dosimetry community's ability to use a consistent set of activation data to derive consistent spectral characterizations. The ACRR is a pulse and steady-state, pool type research reactor that is commonly used to perform irradiation testing. All three of these configurations and their corresponding REAL-201X spectra and measured activities are to be adjusted using the `glmnet` package. It was determined in all cases that it was beneficial to instead use the elastic net regression and optimize for the alpha parameter that bridges between the LASSO and ridge regressions.

Free field environment

The free field environment of the ACRR is configured such that the 23.3 cm diameter central cavity is unoccupied. Within the central cavity, the ACRR maintains an epithermal neutron fluence spectrum. The published benchmark contains a calculated spectrum in the SAND 640 group structure and the measured activities for 31 reactions resulting from 21 different activation foils irradiated in the free field environment of the ACRR central cavity. [3] The high fidelity calculations of the

calculated neutron spectrum were performed with MCNP by Vega, et al., and the resulting model of the ACRR in the free field calculation is shown in Figure III.1.

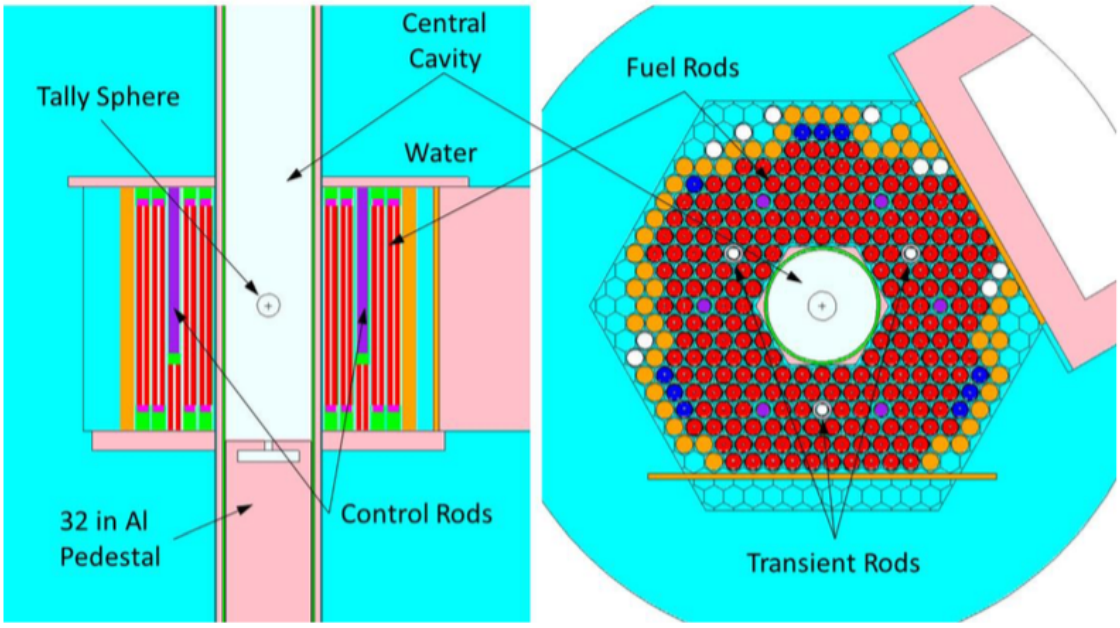

Fig. III.1. MCNP model by Vega, et al., of the ACRR and the central cavity in the free field configuration.

Of the 31 reactions, 23 were chosen to be used in the model. The remaining 8 were chosen not to be in the model due to significant discrepancies between the measured reaction probabilities and the calculated reaction probabilities with the simulated spectrum. The Ni-58 (n,p) reaction was chosen as the reference reaction. The forward difference approximation as detailed in the previous chapter was used to optimize for the greatest continuity in the adjusted spectrum. This optimization resulted in an elastic net alpha parameter of 0.3. The resulting adjustments to the spectrum are shown in Figure III.2. Table III.1 shows the measured and calculated reaction probabilities for each reaction.

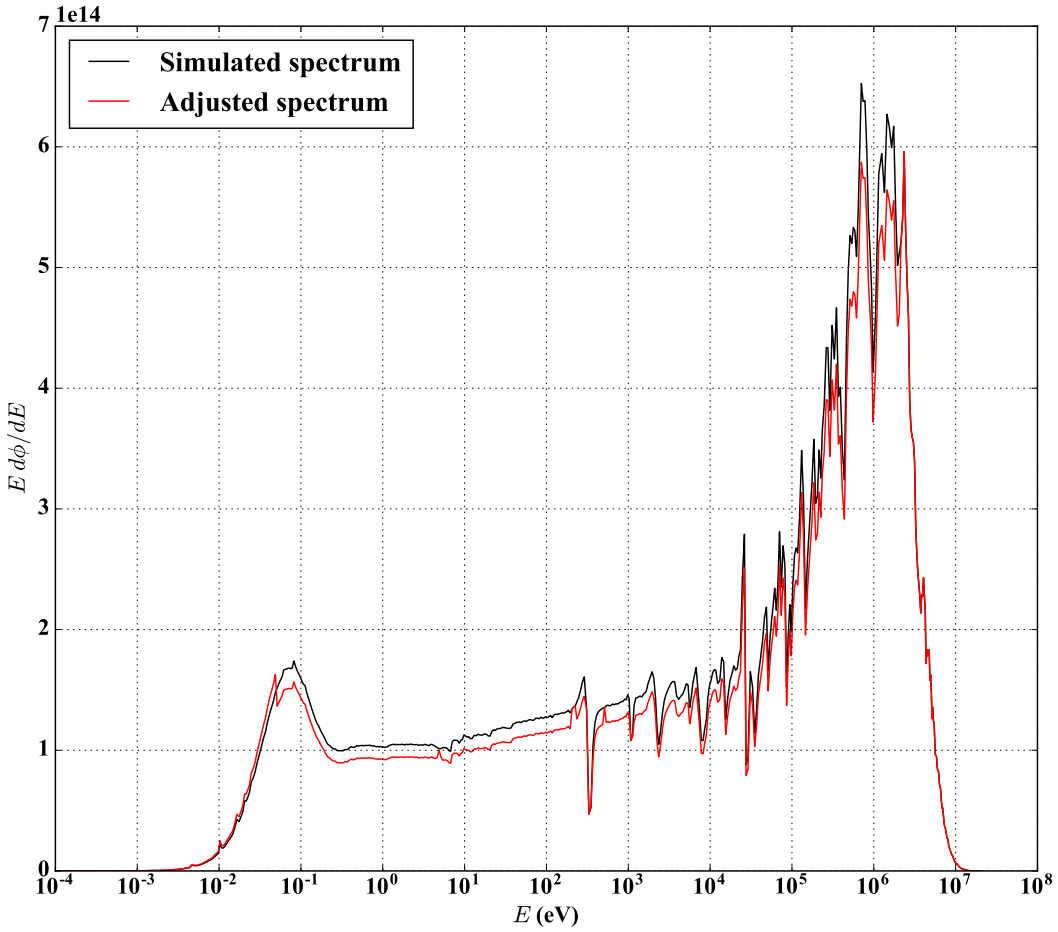

Fig. III.2. The adjustment performed on the MCNP calculated spectrum for the ACRR configured in the free field environment.

Table III.1: The measured activation foil probabilities used in the ACRR free field environment and the resulting change in probabilities due to the spectrum adjustment.

Nuclide	Reaction	Product	Cover	Measured RP	Adjusted RP	Percent Difference
Ni-58	(n,p)	Co-58	Bare	8.641E-11	8.579E-11	0.731
Mg-24	(n,p)	Na-24	Bare	1.002E-12	1.048E-12	4.500
S-32	(n,p)	P-32	Bare	5.246E-11	5.458E-11	3.959
Ti-46	(n,p)	Sc-46	Bare	8.202E-12	8.392E-12	2.289
Ti-47	(n,p)	Sc-47	Bare	1.589E-11	1.518E-11	4.547
Ti-48	(n,p)	Sc-48	Bare	2.007E-13	2.121E-13	5.517
Fe-54	(n,p)	Mn-54	Bare	6.600E-11	6.299E-11	4.601
Fe-56	(n,p)	Mn-56	Bare	7.784E-13	7.793E-13	0.114
Co-59	(n,p)	Fe-59	Bare	9.965E-13	1.042E-12	4.510
Ni-60	(n,p)	Co-60	Bare	1.517E-12	1.570E-12	3.341
Zn-64	(n,p)	Cu-64	Bare	3.164E-11	3.030E-11	4.332
Zr-90	(n,2n)	Zr-89	Bare	6.971E-14	6.534E-14	6.463
Nb-93	(n,2n)	Nb-92m	Bare	3.223E-13	3.044E-13	5.706
Na-23	(n,γ)	Na-24	Bare	1.413E-13	1.687E-10	16.970
Sc-45	(n,γ)	Sc-46	Bare	7.643E-09	8.141E-09	6.309
Mn-55	(n,γ)	Mn-56	Bare	4.636E-09	4.607E-09	0.626
Fe-58	(n,γ)	Fe-59	Bare	4.619E-10	4.554E-10	1.420
Au-197	(n,γ)	Au-198	Bare	1.900E-07	1.813E-07	4.654
Au-197	(n,γ)	Au-198	Cd	1.590E-07	1.680E-07	5.457

LB44 environment

The purpose of the LB44 bucket environment of the ACRR is to fit within the 23.3 diameter of the ACRR central cavity and filter out thermal neutrons and gamma rays within the volume, therefore producing an epithermal neutron environment with a high neutron-to-gamma ray ratio. The LB44 bucket has an outer annular volume filled with B_4 C powder with an inner Pb liner. The published benchmark contains a calculated spectrum in the SAND 640 group structure and the measured activities for 31 reactions resulting from 18 different activation foils irradiated in the LB44 environment of the ACRR central cavity. [4] The high fidelity calculations of the calculated neutron spectrum were performed with MCNP by Vega, et al., and the resulting model of the ACRR in the LB44 environment is shown in Figure III.3.

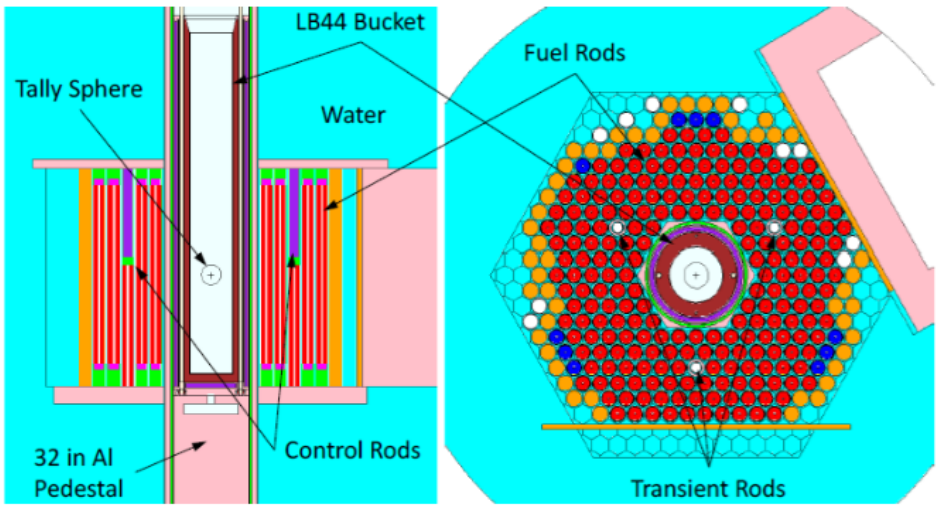

Fig. III.3. MCNP model by Vega, et al., of the ACRR and the central cavity configured in the LB44 environment.

Of the 31 reactions, 17 were chosen to be used in the model. The remaining 14 were chosen not to be in the model due to significant discrepancies between the measured reaction probabilities and the calculated reaction probabilities with the simulated spectrum. The Ni-58 (n,p) reaction was

chosen as the reference reaction. The forward difference approximation as detailed in the previous chapter was used to optimize for the greatest continuity in the adjusted spectrum. This optimization resulted in an elastic net alpha parameter of 0.25. The resulting adjustments to the spectrum are shown in Figure III.4.4. Table III.2 shows the measured and calculated reaction probabilities for each reaction.

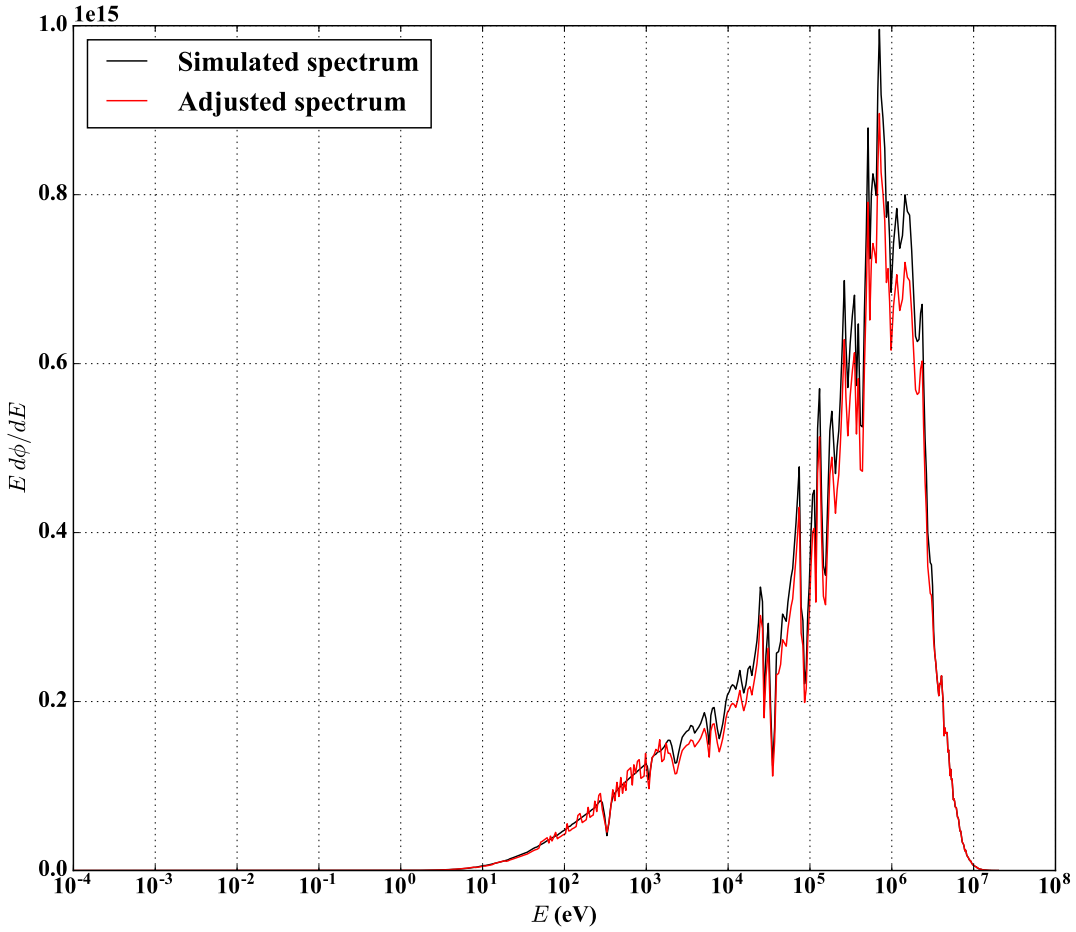

Fig. III.4. The adjustment performed on the MCNP calculated spectrum for the ACRR configured in the LB44 environment.

Table III.2: The measured activation foil probabilities used in the ACRR LB44 environment and the resulting change in probabilities due to the spectrum adjustment.

Nuclide	**Reaction**	**Product**	**Cover**	**Measured RP**	**Adjusted RP**	**Percent Difference**
Ni-58	(n,p)	Co-58	Bare	8.482E-11	8.150E-11	3.994
Co-59	(n,p)	Fe-59	Bare	9.152E-13	9.360E-13	2.334
Ti-46	(n,p)	Sc-46	Bare	7.020E-12	7.514E-12	6.800
Ti-47	(n,p)	Sc-47	Bare	1.510E-11	1.488E-11	1.289
Ti-48	(n,p)	Sc-48	Bare	1.761E-13	1.878E-13	6.421
Fe-54	(n,p)	Mn-54	Bare	5.835E-11	5.898E-11	1.074
Fe-56	(n,p)	Mn-56	Bare	6.413E-13	6.902E-13	7.340
Nb-93	(n,2n)	Nb-92m	Bare	2.563E-13	2.682E-13	4.513
In-115	(n,n')	In-115m	Bare	2.287E-10	1.982E-10	14.278
Mn-55	(n,2n)	Mn-54	Bare	1.361E-13	1.219E-13	11.055
Co-59	(n,2n)	Co-58	Bare	1.104E-13	1.193E-13	7.743
Ni-60	(n,p)	Co-60	Bare	1.296E-12	1.392E-12	7.144
Fe-58	(n,γ)	Fe-59	Bare	6.878E-11	6.531E-11	5.176
Mn-55	(n,γ)	Mn-56	Bare	5.436E-10	5.819E-10	6.803
Cu-63	(n,γ)	Cu-64	Bare	4.387E-10	4.344E-10	0.968
In-115	(n,γ)	In-116m	Bare	2.077E-09	2.673E-09	25.09
Au-197	(n,γ)	Au-198	Bare	8.162E-09	8.00E-09	2.030

PLG environment

The purpose of the Polyethylene-Lead-Graphite (PLG) bucket environment of the ACRR is to fit within the 23.3 cm diameter of the ACRR central cavity and partially thermalize the neutron fluence while attenuating, to some degree, the gamma-ray fluence. This, as a result, produces a larger thermal neutron fluence. The PLG bucket contains an outer annular volume filled with high density polyethylene, an annulus of lead, and an inner annulus of graphite. The published benchmark contains a calculated spectrum in the SAND 640 group structure and the measured activities for 37 reactions resulting from 20 different activation foils irradiated in the PLG-1 environment of the ACRR central cavity. [5] The high fidelity calculations of the calculated neutron spectrum were performed with MCNP by Vega, et al., and the resulting model of the ACRR with the PLG-1 bucket on the 32-inch pedestal is shown in Figure III.5.

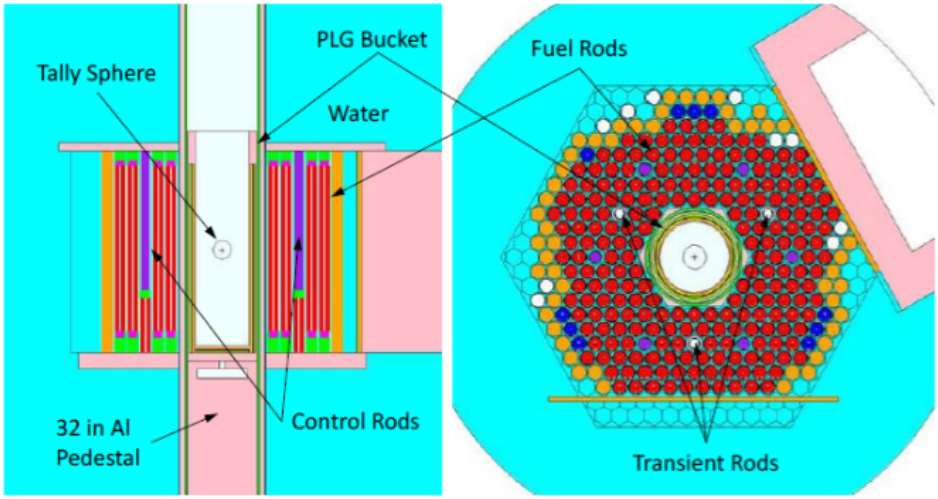

Fig. III.5. MCNP model by Vega, et al., of the ACRR and the central cavity with the PLG-1 bucket on the 32-inch pedestal.

Of the 37 reactions, 25 were chosen to be used in the model. The remaining 12 were chosen not to be in the model due to significant discrepancies between the measured reaction probabilities and

the calculated reaction probabilities with the simulated spectrum. The Ni-58 (n,p) reaction was chosen as the reference reaction. The forward difference approximation as detailed in the previous chapter was used to optimize for the greatest continuity in the adjusted spectrum. This optimization resulted in an elastic net alpha parameter of 0.33. The resulting adjustments to the spectrum are shown in Figure III.6. Table III.3 shows the measured and calculated reaction probabilities for each reaction.

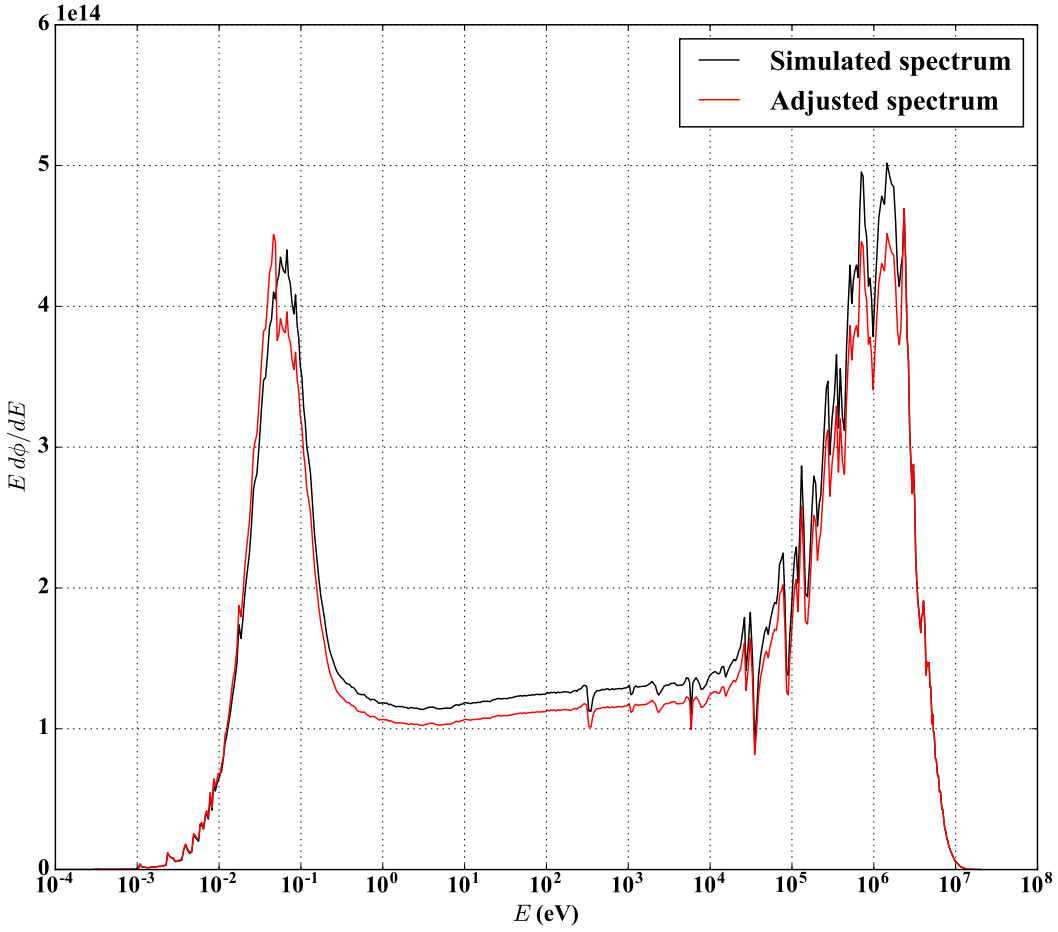

Fig. III.6. The adjustment performed on the MCNP calculated spectrum for the ACRR configured in the PLG environment.

Table III.3: The measured activation foil probabilities used in the ACRR PLG environment and the resulting change in probabilities due to the spectrum adjustment.

Nuclide	Reaction	Product	Cover	Measured RP	Adjusted RP	Percent Difference
Ni-58	(n,p)	Co-58	Bare	6.882E-11	6.881E-11	0.028
Mg-24	(n,p)	Na-24	Bare	7.666E-13	8.469E-13	9.948
S-32	(n,p)	P-32	Bare	4.058E-11	4.347E-11	6.870
Ti-46	(n,p)	Sc-46	Bare	6.404E-12	6.741E-12	5.126
Ti-47	(n,p)	Sc-47	Bare	1.265E-11	1.228E-11	2.945
Ti-48	(n,p)	Sc-48	Bare	1.627E-13	1.710E-13	4.958
Mn-55	(n,2n)	Mn-54	Bare	1.373E-13	1.090E-13	23.064
Fe-54	(n,p)	Mn-54	Bare	4.980E-11	5.031E-11	1.021
Fe-56	(n,p)	Mn-56	Bare	6.186E-13	6.283E-13	1.551
Co-59	(n,p)	Fe-59	Bare	8.154E-13	8.363E-13	2.534
Co-59	(n,2n)	Co-58	Bare	1.122E-13	1.066E-13	5.081
Ni-58	(n,2n)	Ni-57	Bare	2.126E-15	2.121E-15	0.024
Ni-60	(n,p)	Co-60	Bare	1.253E-12	1.265E-12	0.968
Cu-63	(n,α)	Co-60	Bare	3.599E-13	3.073E-13	15.786
Zn-64	(n,p)	Cu-64	Bare	2.447E-11	2.412E-11	1.443
Zr-90	(n,2n)	Zr-89	Bare	5.747E-14	5.139E-14	11.173
Nb-93	(n,2n)	Nb-92m	Bare	2.411E-13	2.438E-13	1.084
In-115	(n,n')	In-115m	Bare	1.649E-10	1.504E-10	9.181
Na-23	(n,γ)	Na-24	Bare	3.009E-10	4.062E-10	29.788
Sc-45	(n,γ)	Sc-46	Bare	1.734E-08	2.031E-08	16.157
Fe-48	(n,γ)	Fe-59	Bare	9.319E-10	1.066E-09	13.39
Co-59	(n,γ)	Co-60	Bare	2.736E-08	3.528E-08	25.300
Cu-63	(n,γ)	Cu-64	Bare	3.305E-09	3.340E-09	12.344
Au-197	(n,γ)	Au-198	Bare	2.405E-07	2.402E-07	0.122
Au-197	(n,γ)	Au-198	Cd	1.787E-07	1.970E-07	9.719

CHAPTER IV

CONCLUSION

In summary, the linear model that relates a simulated neutron spectrum and measured reaction probabilities was used to adjust the neutron spectrum of the ACRR in three configured environments. The original intent was to use exclusively the LASSO regression, but it was determined that a more viable solution could be obtained using the elastic net regression that bridges the LASSO and ridge regressions.

The resulting adjusted spectra were reasonable adjustments that contained acceptable continuity and characteristics. As this investigation was intended to only be a proof of concept in the use of the LASSO (and later elastic net) regression, it can be concluded that the investigation was a success. It is important to note that in all cases, the elastic net alpha parameter was relatively small ($\alpha \leq 0.4$), which implies that this proof of concept is also dependent on the ridge regression.

There is great difficulty in quantifying the results of such an investigation without accounting for the uncertainty in both the simulated spectrum and the irradiation measurements. With all spectrum unfolding methods that use a large number of energy groups, the fact that there are infinitely many solutions magnifies the problem. Because of this, significantly more work is necessary to quantify the results and optimize the regressions used.

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